# Effects of geometrical parmeters on the performance of Rogowski coil for current measuring

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***Abstract*—Current measuring is the core precondition for relay protection, energy metering and Rogowski coil is one of the most favorable transformers. In this paper, the effects of geometrical parameters of Rogowski coil on its performance are calculated and compared to provide a general overview of geometrical selection. The expressions of mutual inductance of rectangular, circular and oval winding coil are derived and calculated, and the reliabilities of each shape of the coil are compared by the simulation. The results show that in condition of equal section area, the mutual inductance of rectangular, circular and oval winding coil are in descending sequences and the relative error of oval winding coil is the smallest, which is helpful for obtaining accurate current values. The research provides important guidance for the rapid and reasonable geometrical selection of Rogowski coil in engineering applications.**

***Index Terms*—Rogowski coil, geometrical parameters, mutual inductance, reliability, current measuring.**

## INTRODUCTION *(HEADING 1)*

The current measuring and its accuracy is always the core part for relay protection and energy metering [1]. In recent years, with the obvious improvement of rating voltage and transmission capacitance, the conventional ferromagnetic transformers which can not detect and quantify the current pulses accurately, are to be replaced by the transformers with the following requirements such as: cost effective, flexibility of installation, suitable for online non-intrusive measurements and higher sensitivity[2-4]. In the past decades, a lot of current transformers satisfying these conditions have sprung out and been investigated both home and abroad. Among all the suitable transformers, the Rogowski coil is the primary current transformer with respect to all the requirements[5]. In the past decades, many researchers have carried out the effects of geometrical parameters on the performance of Rogowski coil. Wang has devised spread spectrum Rogowski coil composed of active integrator, the passive integrator, the high-pass filter and high-frequency self-integral circuit, whose bandwidth of measurable frequency range is 30Hz to 3.23MHz, and the sensitivity is up to 100mV/A[6]. Tian discussed the errors, interferences and dealing of Rogowski coil electronic current transducer and proved that the return wire is of very efficient anti-interference[7]. Xie studied the transfer function of Rogowski transducer based on the principle and proved the validity and practicability of design method by the testing[8]. In order to measure both steady and transient current, Zhang designed an integral circuit for Rogowski coil based on integrator analysis, and the simulation carried out showed that the design suppressed the interference of integrator zero drift with the sensor[9]. The aforementioned studies mainly cares about the transfer function of Rogowski coil or its measuring error, while in fact the mutual inductance and the relative errors of Rogowski coil varies great with the its geometrical parameters, and such investigation on the geometrical influences are rare both home and abroad. In this paper, the mutual inductance expressions as well as their error of rectangular, circular and oval winding coil are analyzed and derived, and the simulation is carried out to testify the theoretical analysis. The results are important guidance for the geometrical design of Rogowski coil.

## MUTUAL INDUCTANCE OF DIFFERENT SHAPES OF ROGOWSKI COIL

The Rogowski coil is composed by the enameled wire even winding on ceramic or other non-ferromagnetic frame whose differential permeability is close to that of the air[10]. The frame is usually circular and the magnetic flux through each turn is equal, which provide great convenience for both theoretical analysis and numerical calculation. For the circular coil, the shape of the cross-section can be rectangular, circular and oval, while their mutual inductance and the error vary great for the direct influences of structural parameters on Rogowski coil. To further analyze the influences of geometrical parameter on the mutual inductance and obtain the optimal selection of the coil, it is necessary to carry out the relationship between structural parameters and the performance of Rogowski coil.

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### A. Measuring Principle of Rogowski coil

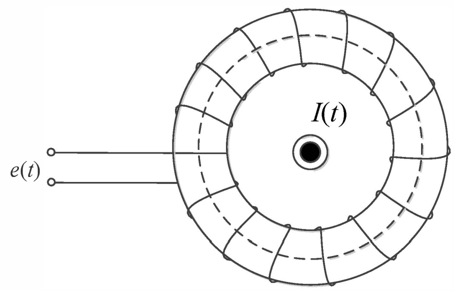


Fig. 1. Measuring principle of Rogowski coil

The measuring principle of Rogowski coil is shown in Fig.1. According to the Ampere Circuit Rule, the equations of magnetic flux can be obtained as follows[11]:

$$\begin{cases} \oint_l Hdl = I \\ B = \mu_0 H \\ \psi = N\iint_S BdS \end{cases} \tag{1}$$

where $H$, $B$ are the magnetic field intensity and density at some defined point and $\psi$ is the sum of magnetic flux of the coil. For the point $r$ away from the center of the coil, the magnetic field density is:

$$B = \mu_0 H = \frac{\mu_0 I}{2\pi r} \tag{2}$$

The induction electromotive force generated from the coil would be[12]:

$$e = -\frac{d\psi}{dt} \tag{3}$$

### B. Rectangular Cross-section Coil

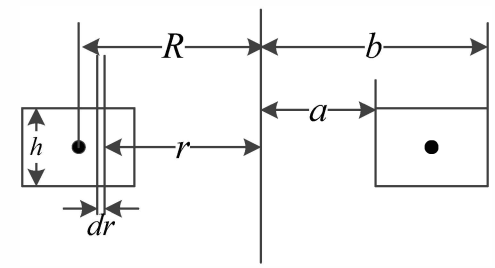


Fig. 2. Diagram of rectangular cross-section coil

As illustrated in Fig. 2, the inner and outer radius are $a$ and $b$, respectively. For the rectangular cross-section coil whose height is $h$, the sum of magnetic flux is:

$$\psi_r = N\iint_S BdS = N\int_a^b \frac{\mu_0 Ih}{2\pi r}dr = \frac{N\mu_0 Ih}{2\pi}\ln\frac{b}{a} \tag{4}$$

The induction electromotive force is calculated as:

$$e_r = -\frac{d\psi_r}{dt} = -\frac{N\mu_0 h}{2\pi}\ln\frac{b}{a}\frac{dI}{dt} \tag{5}$$

Assume that $S$ is the area of the cross-section and $e = h(b-a)$, the mutual inductance of the coil is :

$$M_r = \frac{N\mu_0 h}{2\pi}\ln\frac{b}{a} = \frac{N\mu_0 S}{2\pi(b-a)}\ln\frac{b}{a} \tag{6}$$

The relative error of mutual inductance for the rectangular cross-section coil is:

$$\delta_r = \frac{M_r - M}{M} = \frac{\frac{N\mu_0 S_r}{2\pi(b-a)}\ln\frac{b}{a} - \frac{2N\mu_0(b-a)h}{2\pi(b+a)}}{\frac{2N\mu_0(b-a)h}{2\pi(b+a)}}$$
$$= \frac{(b+a)\ln\frac{b}{a}}{2(b-a)} - 1 = \frac{\left(1+\frac{b}{a}\right)\ln\frac{b}{a}}{2\left(1-\frac{b}{a}\right)} - 1 \tag{7}$$

The equation above indicates that the relative error of mutual inductance has no relationship with the height of the coil, and is only related with the ratio of the outer and inner radius.

### C. Circular Cross-section Coil

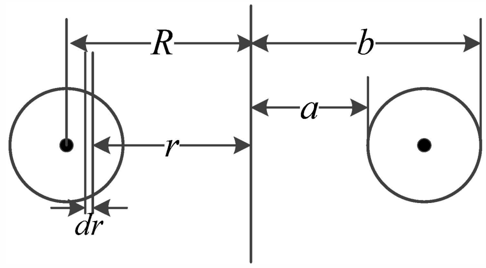


Fig. 3. Diagram of circular cross-section coil

Fig. 3 shows the diagram of circular cross-section coil, whose sum of magnetic flux is:

$$\psi_c = N\left(\int_a^R \frac{2\mu_0 I\sqrt{R^2-(R-r)^2}}{2\pi r}dr + \int_R^b \frac{2\mu_0 I\sqrt{R^2-(r-R)^2}}{2\pi r}dr\right)$$
$$= \mu_0 IN\left[R - \sqrt{R^2 - ((b-a)/2)^2}\right] \tag{8}$$

The induction electromotive force of circular cross-section coil is:

$$e_c = -\frac{d\psi_c}{dt} = -\mu_0 N\left[R - \sqrt{R^2 - ((b-a)/2)^2}\right]\frac{dI}{dt} \tag{9}$$

The mutual inductance of the coil is:

$$M_c = \mu_0 N\left[R - \sqrt{R^2 - ((b-a)/2)^2}\right] \tag{10}$$

For circular cross-section coil, the relative error of mutual inductance can be obtained as:

$$\delta_c = \frac{M_c - M}{M} = \frac{\mu_0 N\left[R-\sqrt{R^2-\left((b-a)/2\right)^2}\right] - \dfrac{\mu_0 N\pi\left((b-a)/2\right)^2}{2\pi R}}{\dfrac{\mu_0 N\pi\left((b-a)/2\right)^2}{2\pi R}} = \frac{2}{1+\sqrt{1-\left((b-a)/(2R)\right)^2}} - 1 \qquad (11)$$

From Fig. 3 the relationship between $R$ and the radius of inner and outer radius of the coil can be found as:

$$R = (b+a)/2 \qquad (12)$$

The relative error of the circular section-cross coil is further obtained as:

$$\delta_c = \frac{2}{1+\sqrt{1-\left(\dfrac{b-a}{b+a}\right)^2}} - 1 \qquad (13)$$

It can be derived from Equation (13) that the relative error is related with the ratio of outer and inner radius, whose trend is similar with that of rectangular coil. When the outer radius is much larger than inner radius, the relative error grows larger; when the two are close to each other, the relative error becomes smaller.

*D. Oval Cross-section Coil*

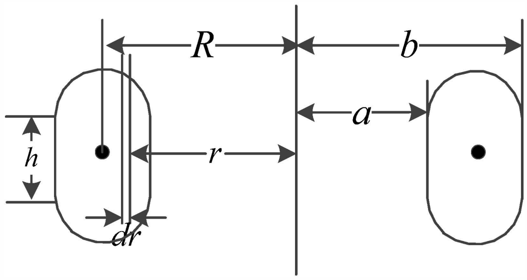


Fig. 4. Diagram of oval cross-section coil

The oval cross-section coil is composed of two semi-circular cross–section with the diameter of $b-a$ and one rectangular cross-section with the size of $(b-a)\times h$ as shown in Fig. 4. Therefore, the sum of magnetic flux is the sum of the both:

$$\psi_o = \psi_r + \psi_c = \frac{\mu_0 NIh}{2\pi}\ln\frac{b}{a} + \mu_0 NI\left[R-\sqrt{R^2-\left((b-a)/2\right)^2}\right] \qquad (14)$$

The induction electromotive force is:

$$e_o = \left\{-\frac{\mu_0 Nh}{2\pi}\ln\frac{b}{a} - \mu_0 N\left[R-\sqrt{R^2-\left((b-a)/2\right)^2}\right]\right\}\frac{dI}{dt} \qquad (15)$$

The mutual inductance of the oval coil can be obtained as:

$$M_o = \frac{\mu_0 Nh}{2\pi}\ln\frac{b}{a} + \mu_0 N\left[R-\sqrt{R^2-\left((b-a)/2\right)^2}\right] \qquad (16)$$

The further relative error of oval coil is:

$$\delta_o = \frac{M_o - M}{M} = \frac{R\ln\frac{b}{a}}{b-a}\,\frac{1}{1+\dfrac{\pi(b-a)}{4h}} + \frac{2}{1+\sqrt{1-\left(\dfrac{b-a}{2R}\right)^2}}\,\frac{1}{1+\dfrac{4h}{\pi(b-a)}} - 1 \qquad (17)$$

Equation (17) indicates that for oval coil, the relative error is also related with the height of the coil, which is different from rectangular and circular coil.

## Simulation And Analysis

*A. Simulation parameters*

To further analyze the influences of different cross-section structural parameters on the performance of Rogowski coil, the simulation is carried out to calculate the mutual inductance and relative error of rectangular, circular and oval cross-section coil. Assume that the loop number of the coil is $N = 500$, the absolute magnetic permeability is $\mu_0 = 4\pi\times10^{-7}\,\mathrm{H}/m$, and the areas of the circular, rectangular and oval cross-section are equal. Since the oval cross-section coil is composed of circular and rectangular parts, assume that the area of the both are identical, and the mutual inductance and relative error of oval cross-section are calculated utilizing the parameters on such assumption.

*B. Keep the distance from the core of cross-section to the center of the coil*

Keep the distance from the core of the cross-section to the center of the coil unchanged, and the distance is $R = 0.3\mathrm{m}$. By changing the diameter of the cross-section $(b-a)$ from $0.01\mathrm{m}$ to $0.4\mathrm{m}$ (with the step of $0.01\mathrm{m}$), the mutual inductance and the relative error of rectangular, circular and oval coil are calculated and analyzed.

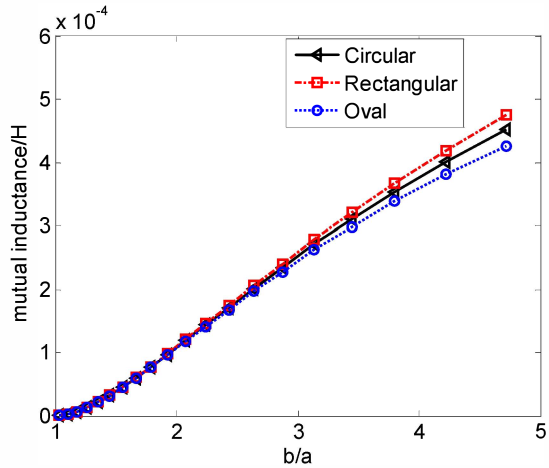


Fig. 5 Mutual inductance of different cross-section coil

Fig. 5 illustrates the mutual inductance of different cross-section coil. The X label is the ratio of outer and inner radius

$b/a$, and the Y label is the mutual inductance of different cross-section coil. In Fig. 5, the mutual inductance of different cross-section coil are mainly identical when the ratio $b/a$ is small( when $b/a$ is smaller than 3.5), and the difference grows larger with the increasing of $b/a$ ( when $b/a$ is larger than 3.5). The ratio of mutual inductance of different coil is shown in Fig. 6 taking the circular cross-section as the reference.

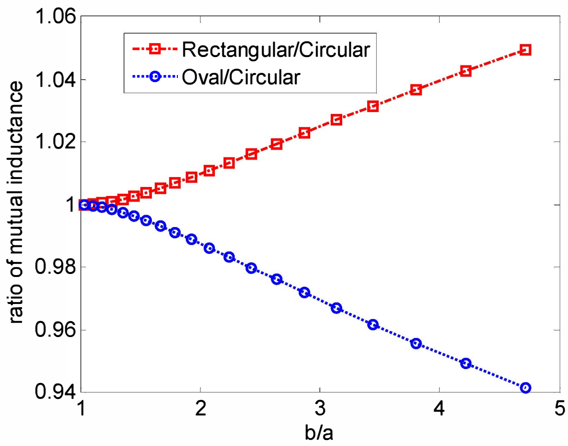


Fig. 6 Diagram of mutual inductance ratio of three cross-section coil

Among the three kinds of coil, the rectangular cross-section coil has the largest mutual inductance, then is the circular cross-section coil, and the oval cross-section coil has the least mutual inductance. The result indicates that to obtain the largest mutual inductance under the circumstance of identical cross-section area, the rectangular cross-section is the best selection.

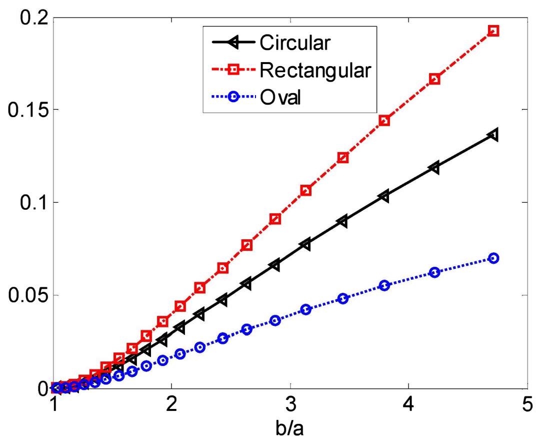


Fig. 7 The relative errors of different cross-section coil

Apart from the value of mutual inductance, the relative error is another factor that affecting the performance of Rogowski coil. The relative errors of three kinds of cross-section coil are calculated and the results are shown in Fig. 7. When the ratio of $b/a$ is smaller than 1.5, the relative errors of three cross-section are mainly the same, approximately within 2%; with the increasing of the ratio, the relative error grows lager and lager, and the relative error of rectangular cross-section coil reaches almost 20% when $b/a$ is 5. View the trend as a whole, the descending order of three cross-section coil is rectangular, circular and oval.

*C. Change the distance from the core of cross-section to the center of the coil*

The former simulation calculates the mutual inductance of three cross-section coil when the distance from the core of the cross-section to the center of the coil keeps unchanged. In this section, the diameter of the cross-section is unchanged, and the distance from the core of cross-section to the center of the coil grows from 0.1m to 0.3m with the step of 0.02m. The areas of three cross-section coil are identical and the mutual inductance is calculated and shown in Fig. 8.

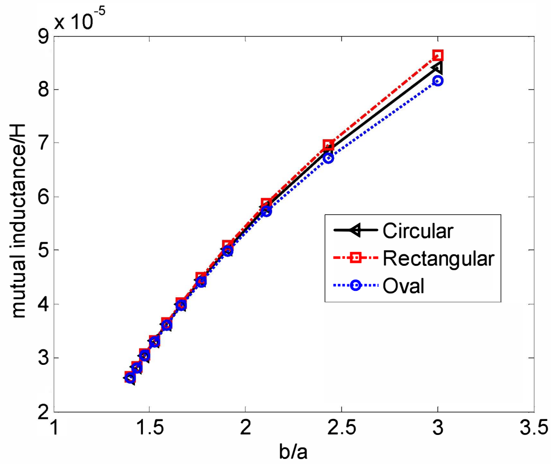


Fig. 8 Mutual inductance of different cross-section coil

It can be seen in Fig. 8 that similar to the result of Fig.5, when changing the distance from the core of cross-section to the center of the coil, the mutual inductance of rectangular cross-section is the largest, then is the circular cross-section, the oval cross-section coil has the least mutual inductance. The ratio of mutual inductance of different coil is shown in Fig. 9 taking the circular cross-section as the reference.

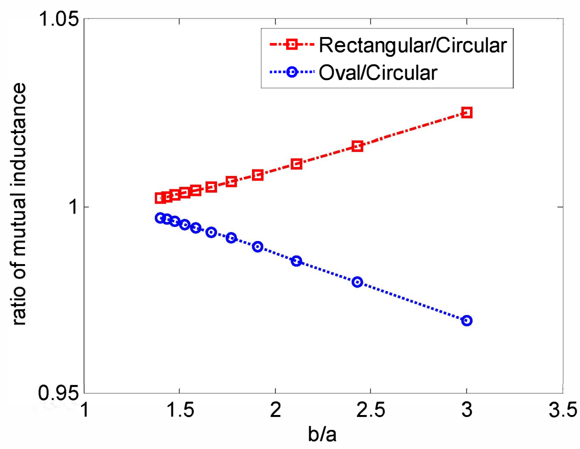

Fig. 9 Diagram of mutual inductance ratio of three cross-section coil

The results in Fig. 9 indicates that the variations of different mutual inductance become obvious with the growing of the ratio $b/a$. Generally, the rectangular to circular mutual inductance increases with the ratio, while the oval to circular mutual inductance decreases with the ratio.

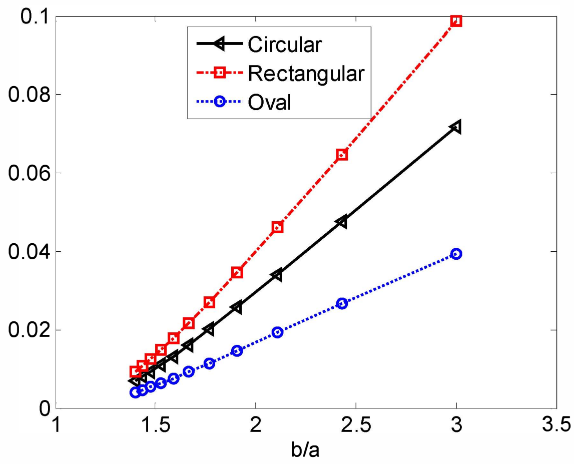


Fig. 10 The relative errors of different cross-section coil

The relative errors of different cross-section coil when changing the distance from the core of cross-section to the center of the coil are calculated and the results are shown in Fig. 10. Obviously, the relative error grows larger with the increasing of the ratio $b/a$. The oval cross-section has the least relative error and its value is 4% when the ratio $b/a$ is 3, while for rectangular the relative error is 10%, this is because the relative error calculated by Eq.(17) is less than Eq.(7) and Eq.(11).

## CONCLUSION

In this paper, the effects of geometrical parameters of Rogowski coil on its performance are calculated from the perspective of mutual inductance and relative error. The expressions of mutual inductance of rectangular, circular and oval cross-section coil are derived and the relative errors are calculated, and the simulation is carried out to investigate the changing law of mutual inductance and relative errors. The results show that the rectangular cross-section coil has the largest mutual inductance, then is the circular cross-section coil, and the oval cross-section coil has the least mutual inductance. Similarly, the oval cross-section coil has the least relative error, which is helpful for obtaining accurate current values. Generally, both the mutual inductance and relative error are proportional to the ratio of outer and inner radius ratio $b/a$.

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